

# STAT 2290 Homework 4 (corrected)

Hand in on April 1 or 2 with your quiz.

## Read these instructions:

- Show your work. Answers without proper justifications receive no credit.
- The only integration techniques we will use are: power rule, substitution rule, exponential functions.

## Types of random variables

### Problem 1.

Let  $X$  be the random variable that records the number of times that "1" comes up when a fair dice is rolled 4 times.

(a) State the values of  $P(X = 0)$ ,  $P(X = 1)$ ,  $P(X = 2)$ ,  $P(X = 3)$ ,  $P(X = 4)$ .

(b) Draw the histogram of the PMF of  $X$ .

(c) Draw the histogram of the CDF of  $X$ .

### Problem 2.

Each of the 10 questions on an exam has multiple choice options A, B, C, D, and E. Let  $X$  be the random variable that records the number of correct answers that result when you randomly guess on each of the 10 questions.

(a) Compute  $P(X \geq 9)$ .

(b) Compute  $F(1)$  if  $F(x)$  is the CDF of  $X$ .

### Problem 3.

You roll a fair dice many times, and you only stop rolling once you get a non-"1" from a dice roll. Let  $Z$  be the random variable that records the number of "1"s you get before you finally get a non-"1" dice roll.

(a) Compute  $f(2)$  if  $f(z)$  is the PMF of  $Z$ .

(b) Compute  $F(2)$  if  $F(z)$  is the CDF of  $Z$ .

(c) Find the probability that it takes at least 3 rolls before you get a non-"1".

(An earlier version of (c) Find the probability that it takes at least 3 rolls before you get a "1".)

### Problem 4.

Consider the Pareto random variable  $X$  whose PDF has the form  $f(x) = c/x^2$  for  $x \geq 1$  and  $f(x) = 0$  otherwise.

(a) Find  $c$ .

(b) Find the CDF of  $X$ .

(c) Find the median of  $X$ .

### Problem 5.

The time to failure (in hours) of a PC component has PDF  $f(x) = \begin{cases} \frac{1}{1000} \exp(-x/1000) & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$ .

(a) Find the CDF  $F(x)$ .

(b) Find the probability that the component lasts more than 1000 hours.

(c) Find the probability that the component fails before 2000 hours.

Note:  $\frac{1}{1000} \exp(-x/1000) = \frac{1}{1000} e^{-x/1000}$

**Problem 6.**

The proportion of people who respond to a mail-order solicitation is a continuous random variable  $X$  that has the probability density function  $f(x) = 2(x + 2)/5$  for  $0 \leq x \leq 1$  and  $f(x) = 0$  elsewhere.

(a) Show that  $P(0 \leq x \leq 1) = 1$ .

(b) Find the probability that more than  $1/4$  but fewer than  $3/4$  of the people contacted will respond to this type of solicitation.

## Expected value and variance

**Problem 7.**

The probability distribution of the discrete random variable,  $X$ , is

$$f(x) = \binom{5}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{5-x}, \quad x = 0, 1, 2, 3, 4, 5.$$

(a) Find the mean of  $X$ .

(a) Find the standard deviation of  $X$ .

(Hint: Is this the PMF of a special type of RV?)

**Problem 8.**

You repeatedly draw cards from a standard deck of 52 cards.

(Imagine you are playing blackjack and you repeatedly "hit".)

Let  $X$  be the random variable that records the number of non-heart cards you receive before finally receiving a heart card.

(a) Find Approximate  $E(X)$  using a geometric random variable.

(b) Find Approximate  $\sigma_X$  using a geometric random variable.

(c) Explain why the geometric random variable only gives an approximate answer to (a) and (b) above.

**Problem 9.**

The proportion of people who respond to a mail-order solicitation is a continuous random variable  $X$  that has the probability density function  $f(x) = 2(x + 2)/5$  for  $0 \leq x \leq 1$  and  $f(x) = 0$  elsewhere.

(a) Find  $E(X)$ .

(b) Find  $\sigma_X$ .

**Problem 10.**

The random variable  $X$  measures the heights in inches of US men between ages 20-29. We know that  $E(X) = 70$  inches and  $\sigma_X = 3$  inches. Re-express  $E(X)$  and  $\sigma_X$  in terms of centimeters.

**Problem 11.**

The random variable  $Y$  measures the average daily temperature of each day in Glassboro in 2025. We know that  $E(Y) = 55^\circ F$  and  $\sigma_Y = 18^\circ F$ . Re-express these in terms of Celsius.